Name:______________________________ Date:______

A2CC: Remainder Theorem and Factor Theorems

Do now:

1. Let $P(x) = 3x^5 + 5x^4 - 4x^3 + 7x + 3$.
   (a) Find the quotient and remainder when $P(x)$ is divided by $x + 2$.
   (b) Find $P(-2)$.

Remainder Theorem:
If the polynomial $P(x)$ is divided by $x - c$, then the remainder is the value $P(c)$.

1. Let $P(x) = x^3 - 2x^2 + 3x - 1$. Find $P(3)$ using 2 different methods.

Factor Theorem:
A polynomial $P(x)$ has a factor of $x - c$ if and only if $P(c) = 0$.

2. Show that $x - 2$ is a factor of $P(x) = x^3 - 3x^2 + 7x - 10$.

3. (a) Use the factor theorem to show that $x+3$ is a factor of $P(x)=x^3-x^2-8x+12$.
   (b) Factor $P(x)$ completely.

4. Let $P(x)=x^3-7x+6$.
   (a) Show that $P(1)=0$.
   (b) Factor $P(x)$ completely.

5. Find a polynomial of degree 4 that has zeros $-3, 0, 1,$ and 5.

THE REMAINDER THEOREM
COMMON CORE ALGEBRA II
EXERCISE SET A

In the last lesson, we saw how two polynomials, when divided, resulted in another polynomial and a remainder. The remainder has a remarkable property in certain types of division. We will explore this relationship in the first exercise.

Exercise #1: Consider each of the following scenarios where we have $\dfrac{p(x)}{x-a}$. In each case, simplify the division using polynomial long division and then evaluate $p(a)$.

(a) $\dfrac{x^2-8x+18}{x-2}$ $\qquad$ $p(x)=x^2-8x+18 \Rightarrow p(2)=$

(b) $\dfrac{x^2-2x-25}{x-7}$ $\qquad$ $p(x)=x^2-2x-25 \Rightarrow p(7)=$

(c) $\dfrac{2x^2+11x+11}{x+3}$ $\qquad$ $p(x)=2x^2+11x+11 \Rightarrow p(-3)=$

(d) $\dfrac{3x^2+7x-20}{x+4}$ $\qquad$ $p(x)=3x^2+7x-20 \Rightarrow p(-4)=$

***Exercise* #2:** If the ratio $\frac{x^2-11x+22}{x-9}$ was placed in the form $q(x)+\frac{r}{x-9}$, where $q(x)$ is a linear function, then which of the following is the value of r?

(1) -3 (3) -9

(2) 5 (4) 4

In the past, the remainder theorem was used primarily to aid in evaluating polynomials. These days it is the primary justification for telling if a linear expression is a factor of a polynomial.

***Exercise* #3:** By definition $(x-a)$ is a factor of $p(x)$ if $\frac{p(x)}{x-a}=q(x)$, where $q(x)$ is another polynomial. What must be true of the remainder, $p(a)$, for $(x-a)$ to be a factor of $p(x)$? Explain.

***Exercise* #4:** Determine if each of the following are factors of the listed polynomials by evaluating the polynomials.

(a) Is $x-3$ a factor of $p(x)=x^2-11x+24$?

(b) Is $x+5$ a factor of $p(x)=2x^2+9x-2$?

(d) Is $x+1$ a factor of $p(x)=x^3-7x^2-11x-3$

(c) Is $x-5$ a factor of $p(x)=x^3-x^2-19x-10$?

***Exercise* #5:** For what value of k will $x-4$ be a factor of $x^2+kx-52$? Show how you arrived at your answer.

THE REMAINDER THEOREM
COMMON CORE ALGEBRA II
Exercise Set B

FLUENCY

1. Which of the following is the remainder when the polynomial x^2-5x+3 is divided by $(x-8)$?

(1) 107 (3) 3

(2) 27 (4) 9

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2. If the ratio $\frac{2x^2+17x+42}{x+5}$ is placed in the form $q(x)+\frac{r}{x+5}$, where $q(x)$ is a polynomial, then which of the following is the correct value of r?

(1) -3 (3) 18

(2) 177 (4) 7

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3. When the polynomial $p(x)$ was divided by the factor $x-7$ the result was $x+\frac{11}{x-7}$. Which of the following is the value of $p(7)$?

(1) -8 (3) 11

(2) 7 (4) It does not exist

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4. Which of the following binomials is a factor of the quadratic $4x^2-35x+24$? Try to do this without factoring but by using the Remainder Theorem.

(1) $x+6$ (3) $x-8$

(2) $x-4$ (4) $x+2$

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5. Which of the following linear expressions is a factor of the cubic polynomial $x^3+9x^2+16x-12$?

(1) $x+6$ (3) $x-3$

(2) $x-1$ (4) $x+2$

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6. Consider the cubic polynomial $p(x)=x^3+x^2-46x+80$.

(a) Using polynomial long division, write the ratio of $\frac{p(x)}{x-3}$ in **quotient-remainder form**, i.e. in the form $q(x)+\frac{r}{x-3}$. Evaluate $p(3)$. How does this help you check your quotient-remainder form?

(b) Evaluate $p(5)$. What does this tell you about the binomial $x-5$?

(c) If $q(x)=\frac{p(x)}{x-5}$, then use polynomial long division to find an expression for the polynomial $q(x)$.

(d) Use your answer from (c) to **completely factor** the cubic polynomial $p(x)$. Besides $x=5$, what are its other zeroes?

7. For the cubic $x^3+7x^2+13x+3$ has only one rational zero, $x=-3$. Use polynomial long division to show that the remainder is zero when dividing the cubic by $x+3$. Then use the quadratic formula to find the other two (irrational) zeroes.